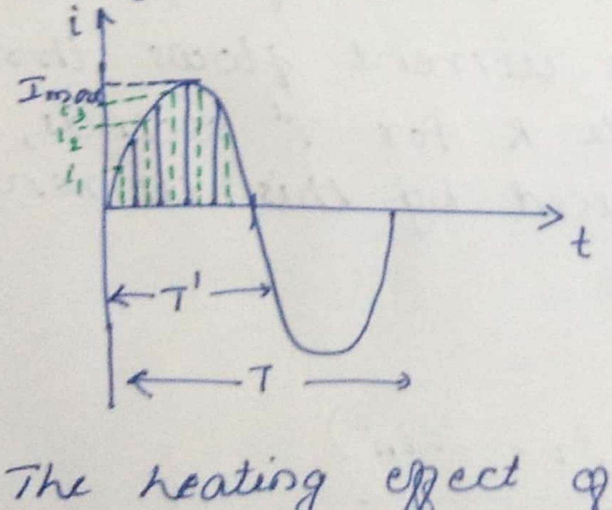
Root mean square value (Rms) or effective value

~~RMS~~ rms value of an alternating current is defined as that equivalent steady state direct current which when flowing through a given resistance for a given time produces the same amount of heat energy as produced by the alternating current when flowing through the same resistance for the same time

two methods — mid ordinate method (graphical method)
— Analytical method (Integral method)

graphical method

It can be employed for both sinusoidal and non sinusoidal waves.



The heating effect of current is proportional to the square of the current. since negative half wave is a repetition of the positive half we need to consider one half only. Let the duration of time be T' sec.

Divide the interval into a large no. of n equal parts each of duration T'/n sec. draw mid ordinate to each. Let the values of mid ordinate be $i_1, i_2, i_3 \cdots i_n$ amp.

If ~~a.c~~ current flows through a resistance of $R\ \Omega$.

heat energy produced in first interval

$$= i_1^2 R \frac{T'}{n} \text{ joule.}$$

heat energy produced in second interval

$$= i_2^2 R \frac{T'}{n} \text{ joule}$$

$ly for 3rd, 4th ~~etc~~ ... nth

Total heat energy produced in all the n intervals

$$W = RT' \frac{(i_1^2 + i_2^2 + \cdots + i_n^2)}{n} \quad \text{— (1)}$$

Let I be the r.m.s value of this alternating current. when the current flows through the same resistance R for T' seconds, then heat energy produced by this current I

$$W = I^2 R T' \quad \text{— (2)}$$

(1) & (2) ⇒

$$I^2 R T' = RT' \frac{(i_1^2 + i_2^2 + \cdots i_n^2)}{n}$$

$$I^2 = \frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}$$ = mean of squares of instantaneous currents

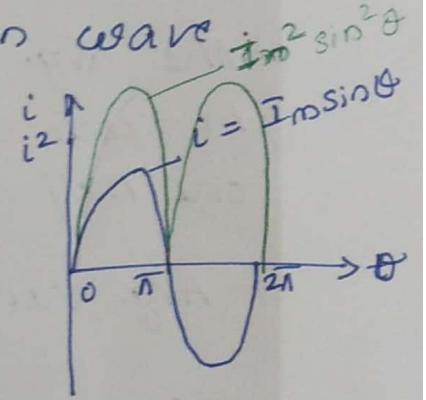
$$I = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}} \text{ amp} \rightarrow \text{rms}$$

$$\text{lly} \quad E = \sqrt{\frac{e_1^2 + e_2^2 + \cdots + e_n^2}{n}} \text{ volt.}$$

Analytical method

This method can be used for finding the rms value of regular wave forms whose mathematical equations are known.

eg: find the r.m.s value of sin wave



equation of ~~sin wave is~~ sinusoidal current

$$i = I_m \sin\theta$$

eqn. of squared value is

$$i^2 = I_m^2 \sin^2\theta.$$

$$\boxed{\text{r.m.s value} = \sqrt{\frac{\text{area of first half cycle of squared wave}}{\text{base}}}}$$

$$\text{square of r.m.s value} = \frac{\int_0^{\pi} i^2 \, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} (I_m^2 \sin^2\theta)\, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\left[\pi - 0\right] = \frac{I_m^2}{2}$$

$$\text{r.m.s value} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Average value

The avg. value of an alternating current is defined as that steady state direct current which transfers across any ckt. the same amount of charge as is transferred by the alternating current during the same time.

For symmetrical alternating current the avg. value over a complete cycle is zero. In such case avg. value is found by considering one half cycle only.

Avg. value → Mid ordinate method

$$I_{av.} = \frac{(i_1 + i_2 + \dots + i_n)}{n}$$

Avg. value → Analytical method

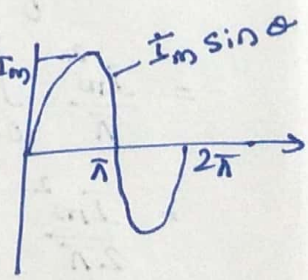
eg: find the avg. value of sine wave.

eqn. of sine wave is

$i = I_m \sin\theta$

$$\boxed{I_{av} = \frac{\text{area of first half cycle}}{\text{base}}}$$

$$I_{av} = \frac{\int_0^{\pi} i \, d\theta}{\pi}$$



$$= \frac{1}{\pi} \int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi} \left[-\cos\theta \right]_0^{\pi} = \frac{I_m}{\pi} [1 - (-1)]$$

$$= \frac{2 I_m}{\pi} = 0.637 \, I_m$$

peak factor (Amplitude factor) k_p

The ratio of max. value to the rms value of an alternating quantity is called its peak factor.

$$\text{peak factor} = \frac{\text{max. value}}{\text{rms value}}$$

Form factor (k_f)

The ratio of rms value to the avg. value of an alternating quantity is called form factor.

$$\text{form factor} = \frac{\text{rms value}}{\text{avg. value}}$$

for sine wave,

$$\text{peak factor} = \frac{I_m}{I_m/\sqrt{2}} = \underline{\underline{1.414}}$$

$$\text{form factor} = \frac{\frac{I_m}{\sqrt{2}}}{\frac{2I_m}{\pi}} = \frac{\cancel{I_m}}{\sqrt{2}} \times \frac{\pi}{2\cancel{I_m}}$$

$$\underline{\underline{= 1.11}}$$

Q.1 An alternating current is given by $i = 62.35 \sin 323t$ A. Determine 1) max. value 2) frequency 3) rms value 4) avg. value 5) form factor.

$i = 62.35 \sin 323t$

$i = I_m \sin \omega t$

$I_m = 62.35$

$\omega = 323$

1) $I_{max} = 62.35$ A

2) $f = \frac{\omega}{2\pi} = \frac{323}{2\pi} = 51.4$ Hz

3) rms value, $I_{rms} = \frac{I_m}{\sqrt{2}}$ → for sine wave $I_{rms} = \frac{I_m}{\sqrt{2}}$ the given eqn is a sine wave

$= \frac{62.35}{\sqrt{2}} = \underline{44.1\ A}$

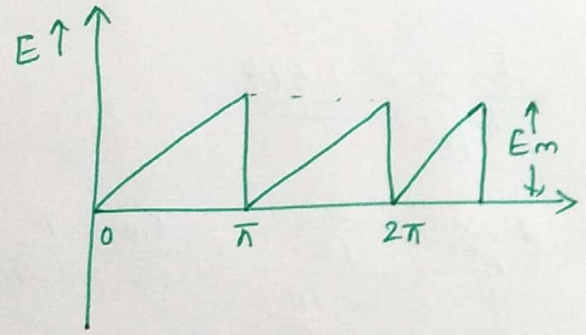
4) $I_{avg.} = \frac{2I_m}{\pi} = \frac{62.35 \times 2}{\pi}$

$= \underline{39.7\ A}$

5) form factor, $k_f = \frac{I_{rms}}{I_{avg}}$

$= \frac{44.1}{39.7} = \underline{1.11}$

2. calculate the average and the rms values of saw tooth voltage wave having max. value E_m voltage. Find also the form factor and peak factor.



Average value $= \frac{\text{area over } 1/2 \text{ cycle}}{\text{base}}$

eqn. of given wave is

$e = m\theta$

where $m = \frac{E_m}{\pi}$.

$e = \frac{E_m}{\pi}\theta$

eqn. of straight line is $y = mx + c$

here $c = 0$.

so $y = mx$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{E_m - 0}{\pi - 0}$

$= \frac{E_m}{\pi}$

area of curve $= \frac{1}{2} Em \times \pi$ $\frac{1}{2}$ base x altitude

π Em

$$E_{av.} = \frac{\frac{1}{2} Em \times \pi}{\pi} = \frac{1}{2} Em = 0.5\, Em$$

$$(E_{rms}^2) = \frac{\text{area of squared wave}}{\text{base}}$$

$$= \frac{1}{\pi} \int_0^\pi e^2 \, d\theta$$

$$= \frac{1}{\pi} \int_0^\pi \frac{Em^2}{\pi^2} \theta^2 \, d\theta = \frac{Em^2}{\pi^3} \left[\frac{\theta^3}{3} \right]_0^\pi$$

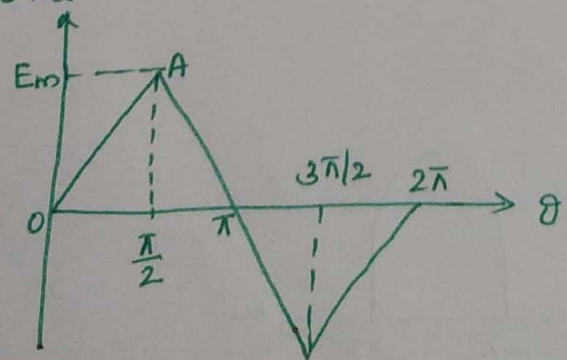
$$= \frac{Em^2}{\pi^3} \cdot \frac{\pi^3}{3} = \frac{Em^2}{3}$$

$$E_{rms} = \sqrt{\frac{Em^2}{3}} = \frac{Em}{\sqrt{3}} = 0.577\, Em$$

$$K_f = \frac{E_{rms}}{E_{avg}} = \frac{0.577\, Em}{0.5\, Em} = 1.1557$$

$$K_p = \frac{Em}{E_{rms}} = \frac{Em}{0.577\, Em} = 1.732$$

3. calculate the average and rms values of the triangular voltage wave having max. value Em volts.



since the wave is symmetrical about $\pi/2$ we have to consider the interval 0 to $\pi/2$ only.

$E_{avg} = \dfrac{\text{area under the curve}}{\text{base}}$

$= \dfrac{\frac{1}{2} E_m \times \frac{\pi}{2}}{\pi/2} = \dfrac{E_m}{2} = \underline{\underline{0.5\ E_m}}$

eqn. of wave OA is

$e = m\theta, \quad m = \dfrac{E_m}{\pi/2} = \dfrac{2E_m}{\pi}$

$e = \dfrac{2E_m}{\pi}\theta.$

$(E_{rms})^2 = \dfrac{\text{area of squared wave}}{\text{base}}$

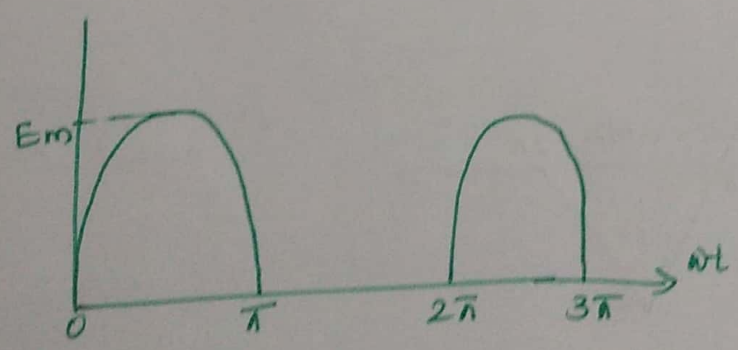
$= \dfrac{2}{\pi}\int_0^{\pi/2} e^2\, d\theta = \dfrac{2}{\pi}\int_0^{\pi/2} 4\dfrac{E_m^2}{\pi^2}\theta^2\, d\theta$

$= \dfrac{2}{\pi}\cdot\dfrac{4E_m^2}{\pi^2}\left[\dfrac{\theta^3}{3}\right]_0^{\pi/2} = \dfrac{8E_m^2}{3\pi^3}\left[\dfrac{\pi}{2}\right]^3$

$= \underline{\underline{\dfrac{E_m^2}{3}}}$

$E_{rms} = \sqrt{\dfrac{E_m^2}{3}} = \dfrac{E_m}{\sqrt{3}} = \underline{\underline{0.577\ E_m}}$

4. Calculate the avg. value, effective value and form factor of o/p voltage wave of half wave rectifier.



Here the wave form repeats only after 2π so the period is from 0 to 2π.

$$E_{avg.} = \frac{1}{2\pi}\left[\int_0^{\pi} E_m \sin\theta \, d\theta + \int_{\pi}^{2\pi} 0 \, d\theta\right]$$

$$= \frac{E_m}{2\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{E_m}{2\pi}\left[1-(-1)\right] = \frac{2E_m}{2\pi} = \frac{E_m}{\pi}$$

The wave eqn. is

$0 < t < \pi$; $e = E_m \sin\theta$

$\pi < t < 2\pi$, $e = 0$

$$(E_{rms})^2 = \frac{1}{2\pi}\int_0^{\pi} E_m^2 \sin^2\theta \, d\theta$$

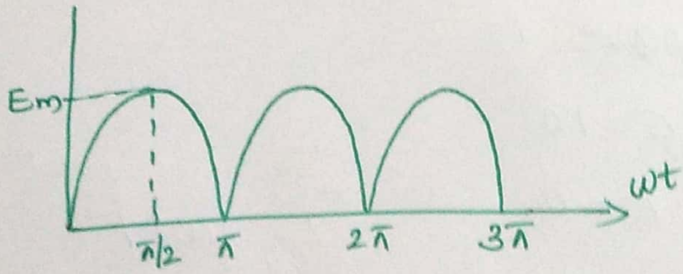
$$= \frac{E_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{E_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi} = \frac{E_m^2}{2\pi} \times \frac{\pi}{2}$$

$$= \frac{E_m^2}{4}$$

$$E_{rms} = \sqrt{\frac{E_m^2}{4}} = \frac{E_m}{2} = 0.5\,E_m$$

5. Find 1) avg. value 2) effective value and 3) form factor of the full wave rectified sine wave.



try to find the answer using base π/2.

$$E_{avg.} = \frac{1}{\pi}\int_0^{\pi} E_m \sin\theta \, d\theta = \frac{E_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{E_m}{\pi}\left[(-1-1)\right] = \frac{2E_m}{\pi} = 0.637\,E_m$$

$$(E_{rms})^2 = \frac{1}{\pi}\int_0^{\pi} E_m^2 \sin^2\theta \, d\theta$$

$$= \frac{E_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

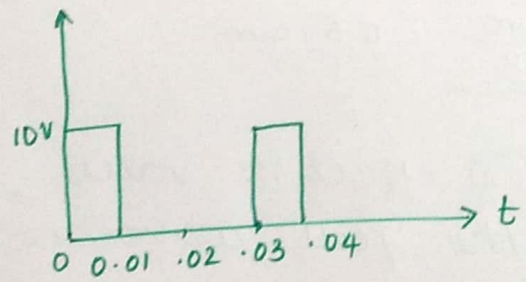
$$= \frac{E_m^2}{2\pi}\left[\theta - \frac{1-\sin 2\theta}{2}\right]_0^{\pi} = \frac{E_m^2}{2\pi}[\pi - 0]$$

$$= \frac{E_m^2}{2}$$

$$E_{rms} = \frac{E_m}{\sqrt{2}} = 0.707 E_m$$

$$K_f = \frac{E_{rms}}{E_{avg}} = \frac{\frac{E_m}{\sqrt{2}}}{2E_m/\pi} = 1.11$$

6. Find the avg. and rms values of the wave form shown.



Here the period is from 0 to 0.03.

eqn. of the wave is

$0 < t < 0.01$; $E = 10$

$0.01 < t < 0.03$; $E = 0$

$$E_{avg.} = \frac{1}{0.03}\int_0^{0.01} 10 \, dt = \frac{1}{0.03}\left[10t\right]_0^{0.01} = \frac{1}{0.03} \times (10 \times 0.01)$$

$$= 3.33V$$

$$E_{rms}^2 = \frac{1}{0.03}\int_0^{0.01} 10^2 \, dt = \frac{1}{0.03} \times \left[10^2 t\right]_0^{0.01}$$

$$= \frac{1}{0.03} \times 100 \times 0.01 = 33.3 V$$

$E_{rms} = \sqrt{33.3} = 5.77V$